Mathematics in the Making

Pattern, Area and Perimeter

Stuart E Bell

Longmans

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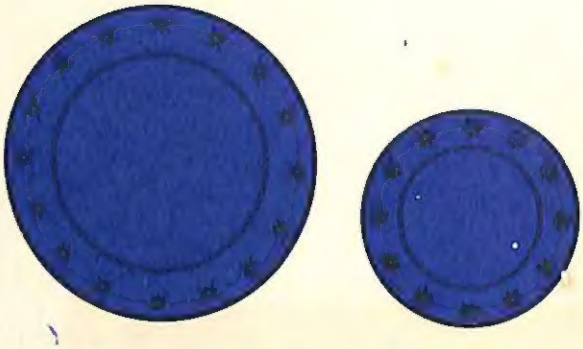
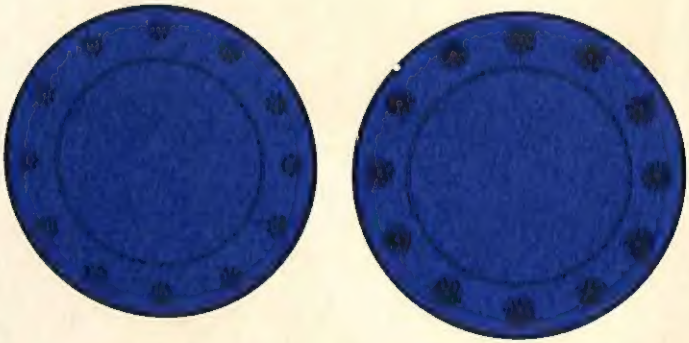
Longmans

Pattern, Area and Perimeter



Which plate holds more ?

You will have no trouble in picking the one on the left.



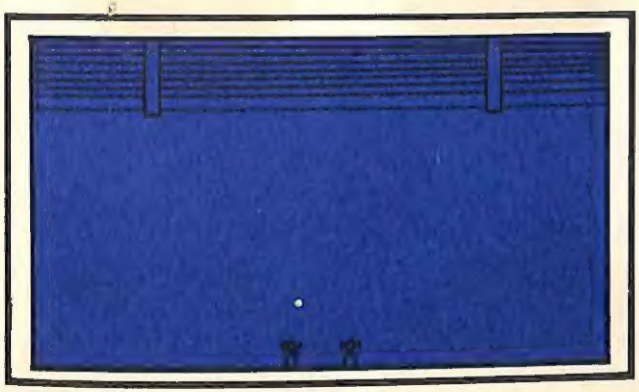
Now look at these plates.

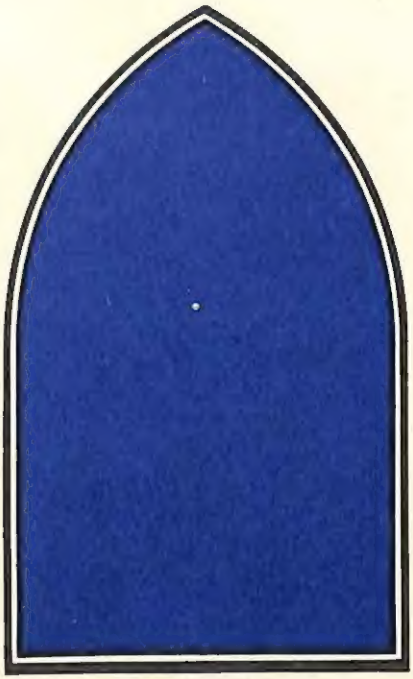
Both plates are the same shape, but it is more difficult to choose the one which will hold more as they are nearly the same size.

Trace the outline of the plate on the left.
Put it over the plate on the right.

Now can you see which is bigger ?

Here are two windows.
One is a church window and the other a kitchen window.





If both were broken, which one would need more glass ?

Trace the shape of the kitchen window and place the tracing over the church window.

Can you see which is bigger ?

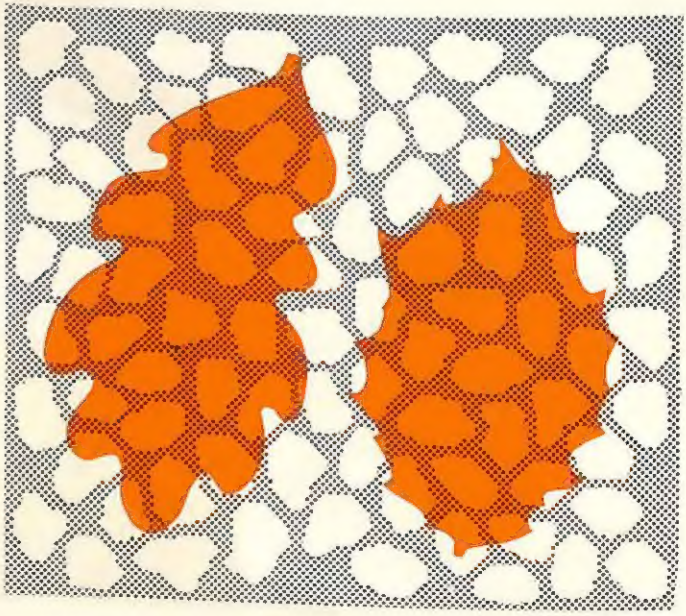
The answer is not easy because the two shapes are not quite the same. Both windows appear to be approximately the same size, but we cannot be sure about this.

Now look at these leaves.

Which one is bigger ?

It is almost impossible to be sure because the leaves are quite different in shape.

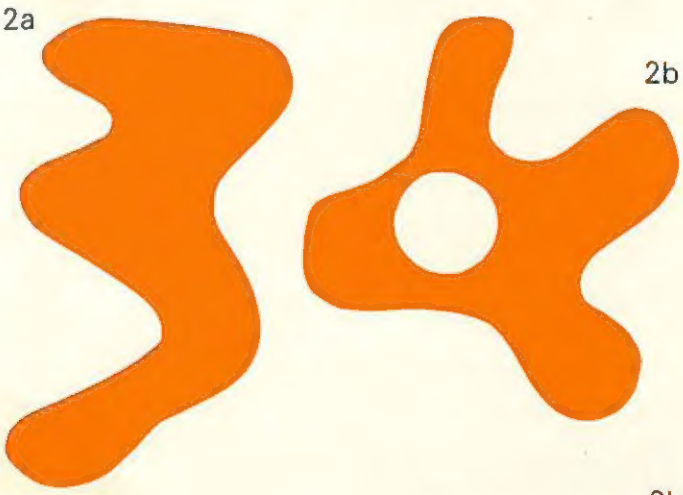
Suppose we fill the leaves with a pattern, like this :

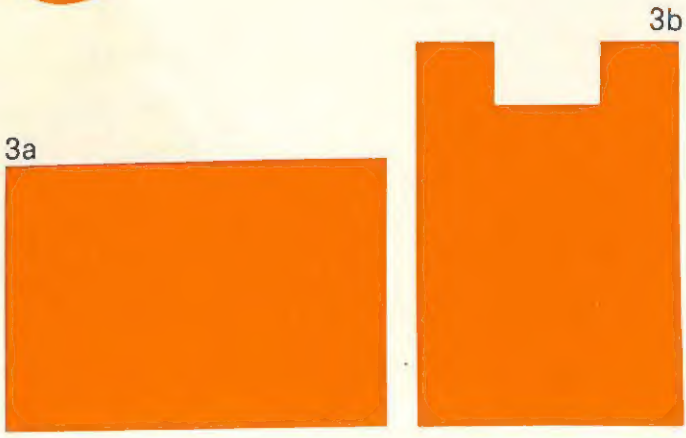


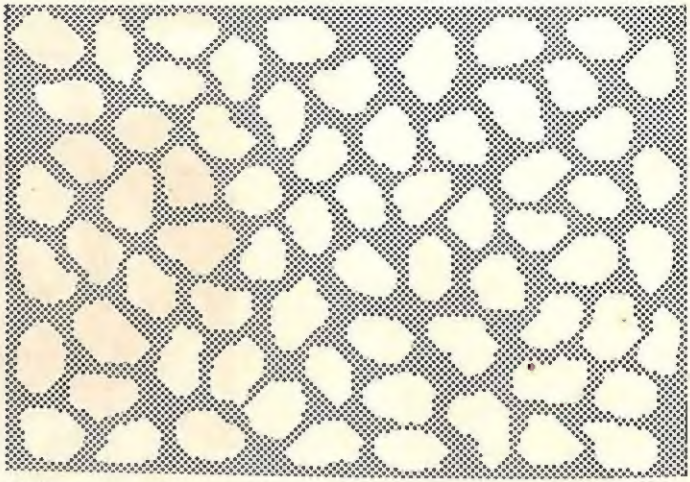
Can you discover a method of finding which leaf is bigger ?

Which is the bigger shape in each of the following pairs?

To find out, trace the shapes and place each one in turn over the pattern on the next page. You will then be able to count the number of shapes as you did when you compared the leaves.



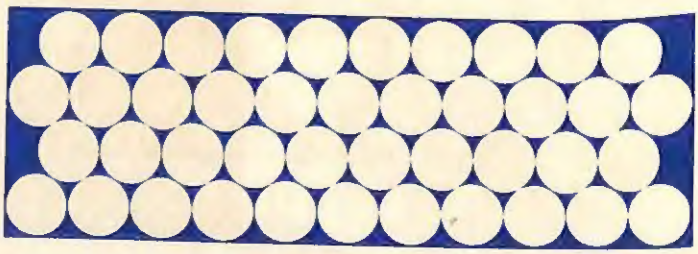




Using a pattern in this way gives us a rough idea how big a shape is and a means of comparing one shape with another.

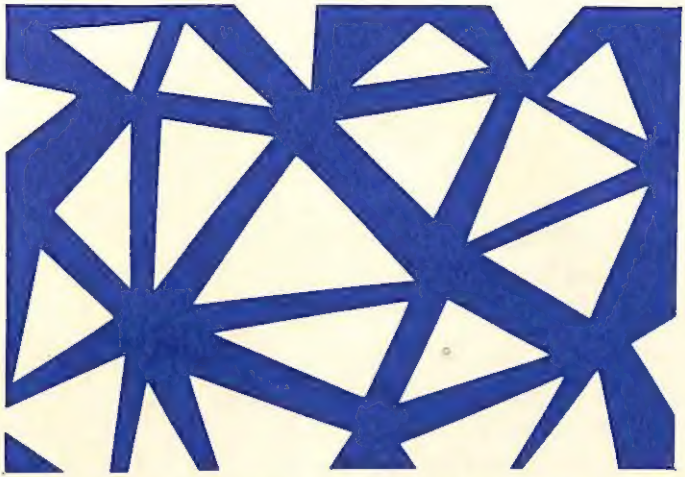
But did you notice that all the different shapes in the pattern were not the same size? You may also have found that there were some corners where no part of the pattern would fit.
Is there a better pattern we could use to compare sizes which would fill all the corners and whose parts are all the same size?

Let us make a pattern of circles, each of which is the same size, like this:



Does this pattern fill up all the space?

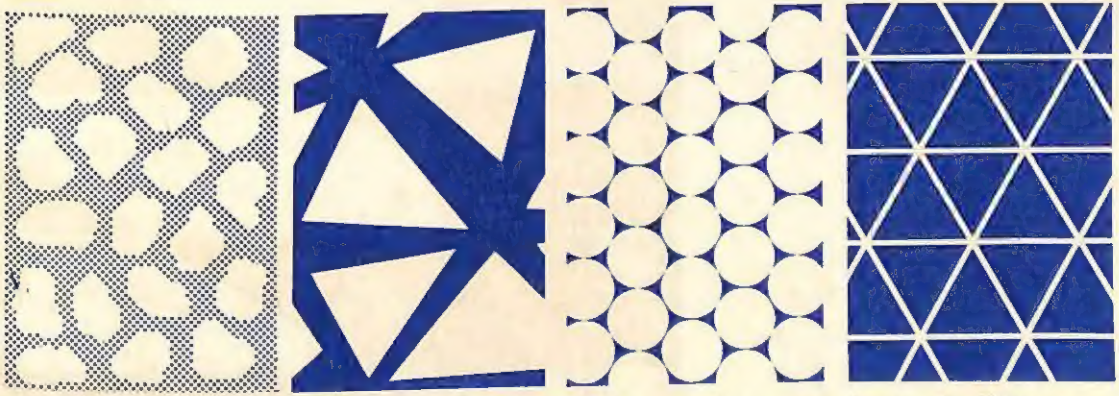
Here is another pattern.



If we were to fit our tracings over this pattern, we could count the number of triangles in each shape.

But are the triangles all the same size ? Do they fill up all the space ?

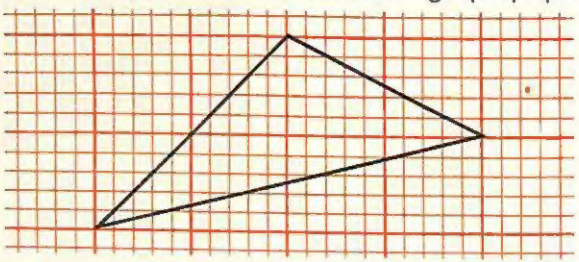
Which pattern fills up a space best of all—the first two patterns, the circles, or the triangles ?



It is clear that some shapes are better for filling a space than others.

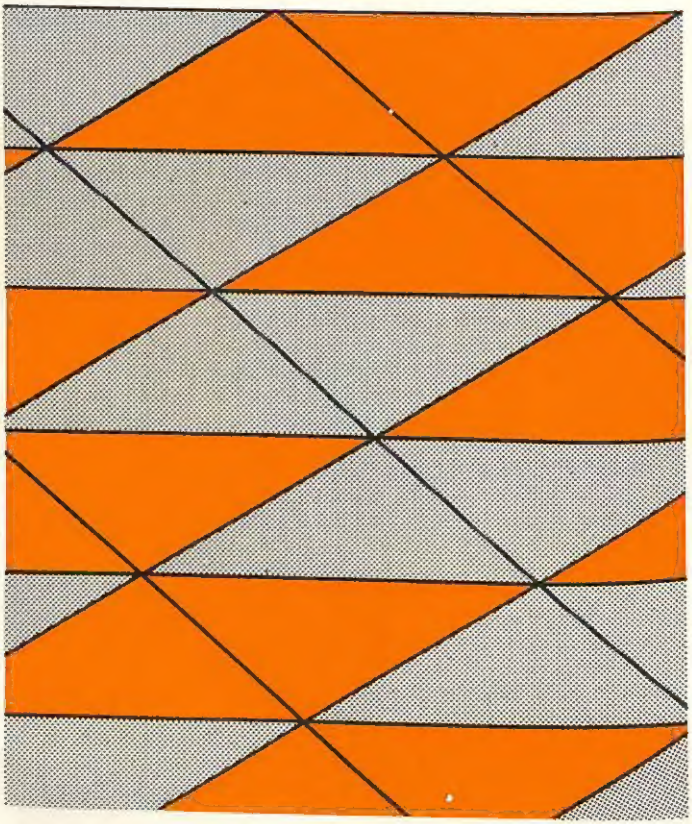
Space-filling shapes

Let us discover other shapes which can be used for making patterns and find which of them are useful for filling up a space.
First of all we will use a triangle.
This triangle has been drawn on graph paper.



Draw and cut out a number of triangles of this size.
If you do not have graph paper, you can trace the triangle.
You will have to cut the triangles out very carefully.

Stick the triangles on to a piece of paper and colour them to make a pattern like this.



Do the triangles fill up all the space ?
Why do you think that these triangles fill up the space and those at the top of page 7 do not ?
Make some more triangles in the same way, and find out whether you can arrange them in a pattern to fill up a space in a different way.

Draw a triangle of a different shape. Make and cut out a number of these. Can you stick them together to fill up a space ?

Look at the pattern of triangles again.
How many sets of lines are there ?
What do you notice about the lines in each set ?

The lines in each set are, of course, the same distance apart.
Lines which are the same distance apart are said to be PARALLEL
Railway lines are parallel, otherwise trains would come off the lines.

Make a list of parallel lines which you can see around you, like this.

PARALLEL LINES

The opposite edges of this book.

By colouring the triangles in the pattern as we did, we made new shapes. How many sides has each of these shapes ?
We call these shapes PARALLELOGRAMS because each pair of opposite sides is parallel.

Are all the parallelograms in the pattern the same size ?
Do they fill up the space ?

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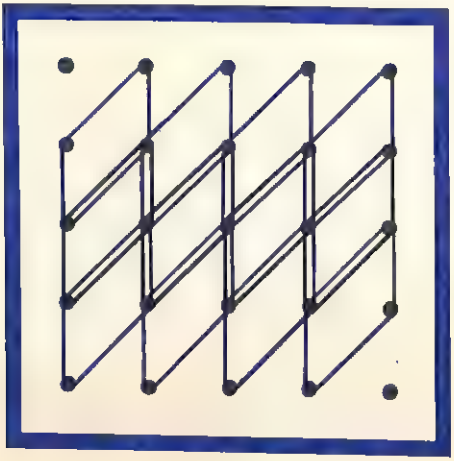
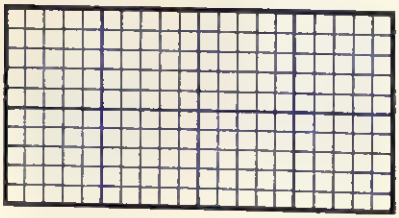
Draw a parallelogram of a different shape on squared paper. Use the shape to make a pattern filling up a space. Colour or decorate the pattern.

An easier way of making a pattern of parallelograms is to use rubber bands on a geo-board or pin-board, like this.



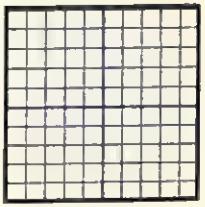
Here is another shape you know well—the OBLONG or RECTANGLE



Draw a number of rectangles all the same shape and size, and cut them out, or make them on the geo-board or pin-board.
Find whether the rectangles can be fitted together to fill up a space.
Can you make more than one pattern with them ?

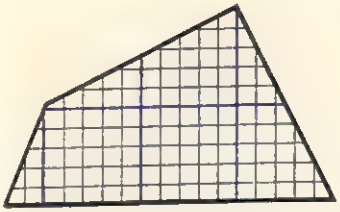
Draw the pattern of rectangles made by the bricklayer when he builds a wall.
In how many other ways does a builder use a pattern of rectangles in building a house ? Draw them.

Now find whether you can use a square in the same way to fill up a space.



Can you think of any patterns of squares which are used in a house to fill up a space ?

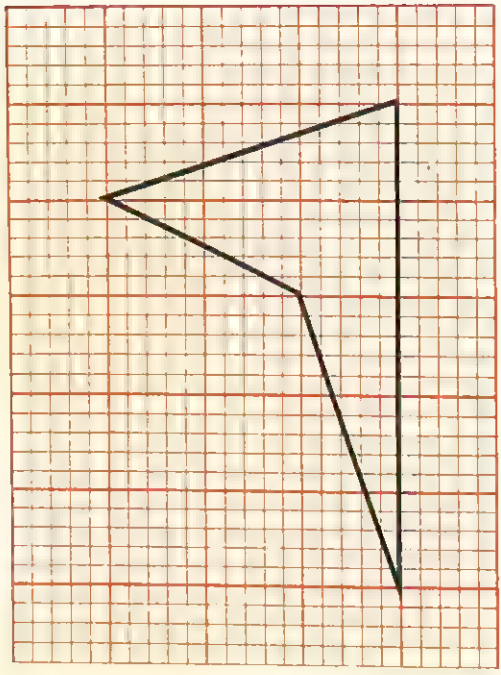
Look carefully at this shape.
How many sides has it ?
Is it a rectangle ? Why not ?
Are the opposite sides parallel ?



All four-sided shapes are called QUADRILATERALS. The parallelogram, square, rectangle and diamond are special kinds of quadrilateral.

Why is this quadrilateral not a parallelogram ?
Draw and cut out a number of quadrilaterals like this one, or make them on the geo-board.
Find whether they can be fitted together to fill a space.
Colour your pattern.

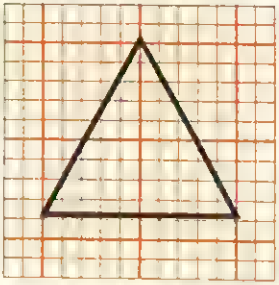
Here is another kind of quadrilateral. Copy it and find whether it can be used to fill up a space. You should be able to make a very attractive pattern.



Make patterns from other quadrilaterals.

This is a special kind of triangle.

What do you notice about its sides ?



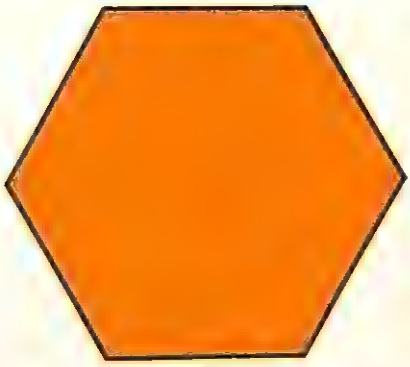
Can you think why ?

We call it an EQUILATERAL TRIANGLE.

Draw and cut out a number of equilateral triangles of the same size.
Can you stick them together to make a shape with six sides ?

Is your hexagon like this ?
If not, make one like it.
Does it look the same if we turn it round ?
Draw and cut out a number of hexagons the same size as the one you have made.
Discover whether they can be fitted together to fill up a space.

Any shape with six sides is called a HEXAGON.



Hexagons make attractive patterns, as you will find if you colour and decorate them.

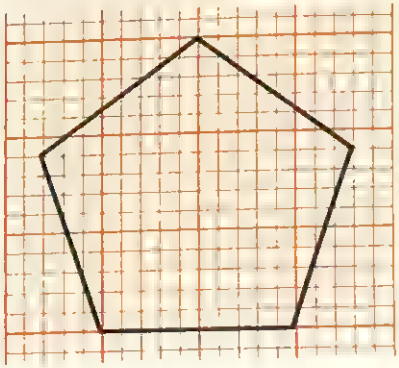
Bees store the honey which they collect in groups of cells called honey combs.

What shape are the cells ?
Can you think of a reason why they are made this shape ?

Look out for other hexagonal shapes.

How many sides has this shape ?

We call it a PENTAGON.



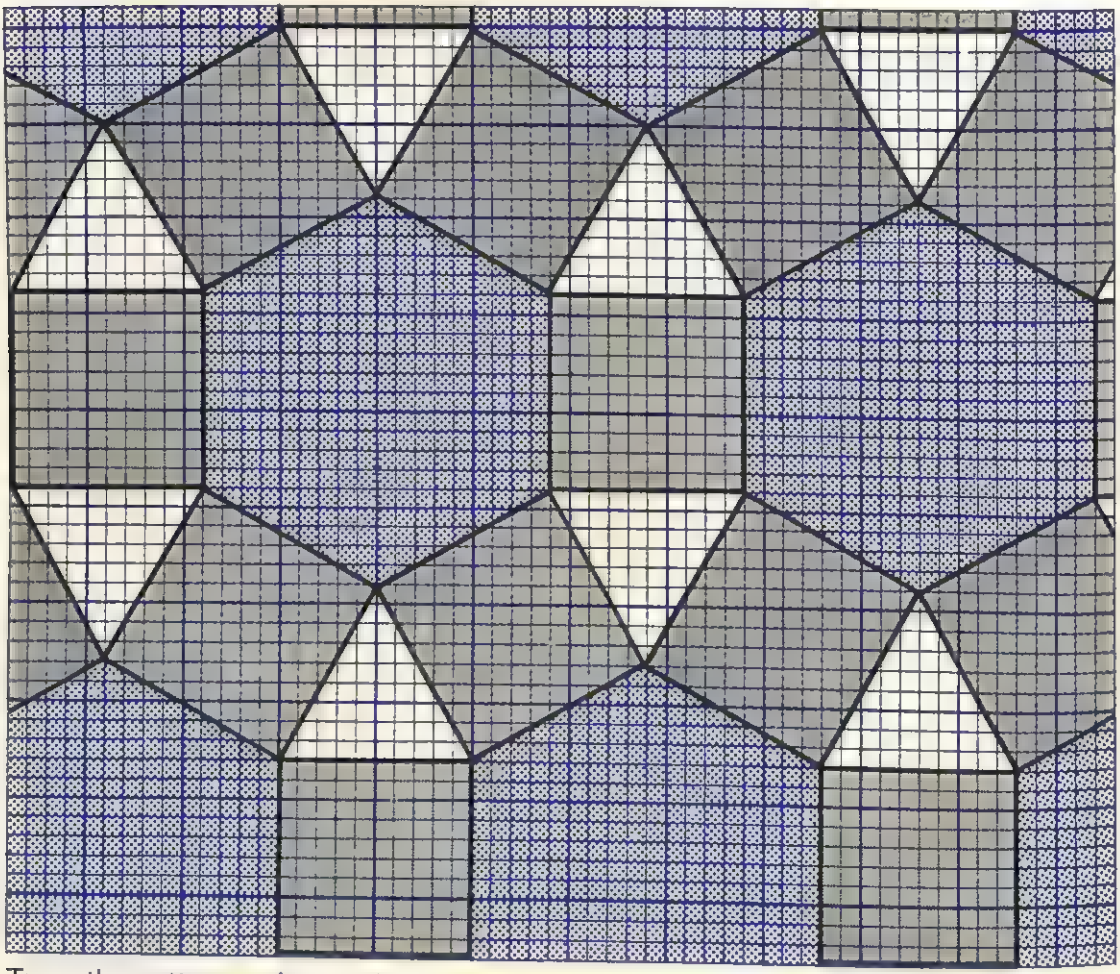
Does it look the same if you turn the page round ?

Using graph paper copy and cut out a number of pentagons of the same size.
Can you use the pentagon to fill up a space ?

How many sides has a threepenny piece? Can you fill up a space with a number of threepenny pieces?

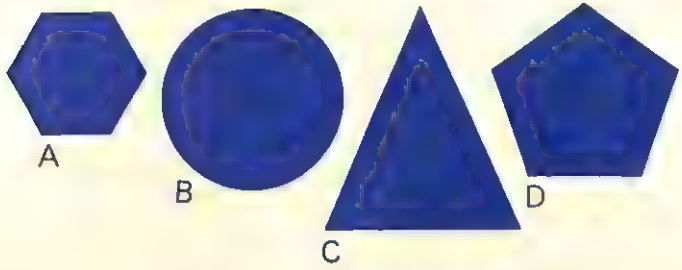
Here is a pattern made up of several shapes, each of which we have found can be used to fill up a space.

What are their names?



Trace the pattern and extend it to fill up a larger space. Colour and decorate it.

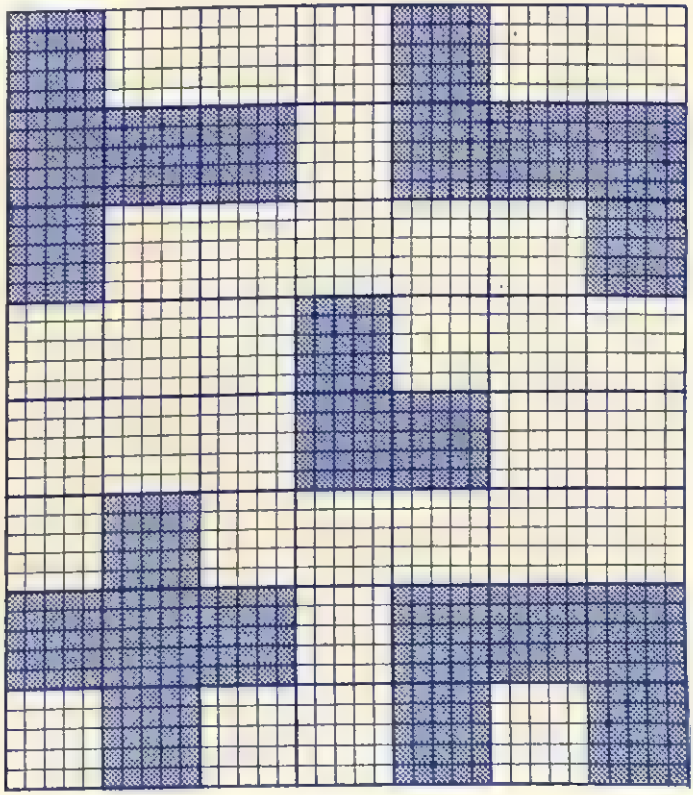
Knowing about shapes which can be used for filling up a space can be very useful to us. For example, suppose we want to fill up a floor with tiles or wood blocks.



Which of these shapes could we use for the tiles or blocks?

Look around you in school, at home, in church, or in the shops, and see which shapes are used to fill spaces.

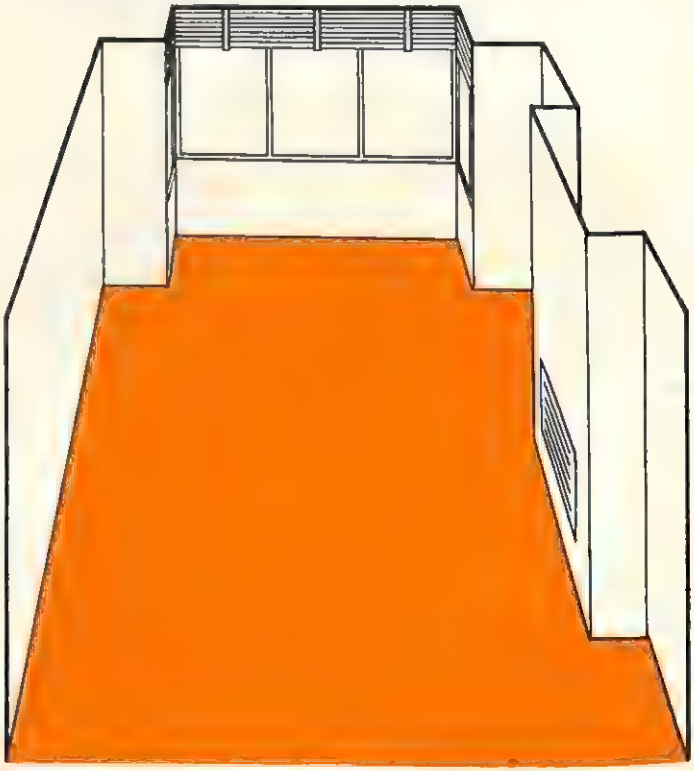
Copy these shapes on to squared or graph paper. Cut out a number of each. Find whether you can fill up a space with each one.



Measuring Areas

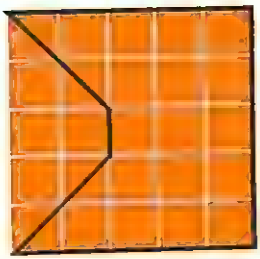
We use the word AREA to describe the size of a surface.
The first plate on page 2 has a bigger area than the second.
We have discovered that we can use shapes such as triangles, rectangles, squares, parallelograms, and hexagons to fill up a space, and that we can compare two spaces by counting the number of shapes in each.

Which of these shapes would be best to use to measure the area of this room?
Think which shape would fit most easily into the corners.



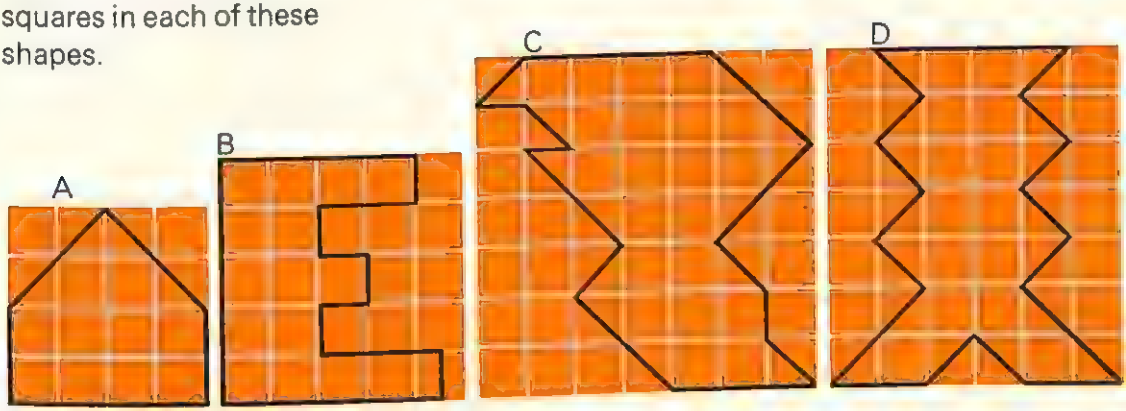
You will probably answer "squares" or "rectangles", because the square corners of these shapes will fit into the square corners in the room. Squares are the shapes we use to count because they have the same length in each direction.

Count the number of squares in this shape.

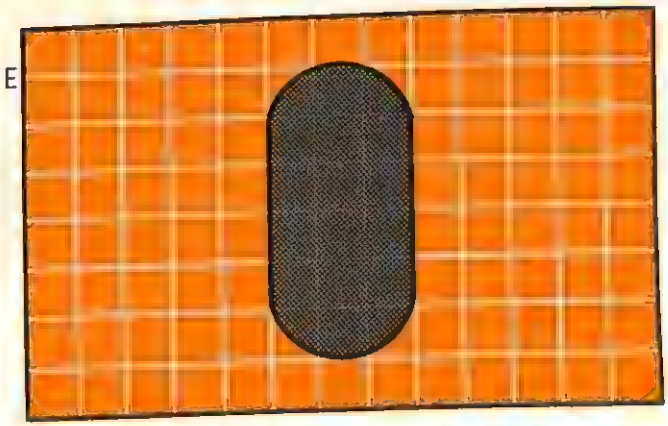


What will you do with the parts of squares?

Count the number of squares in each of these shapes.



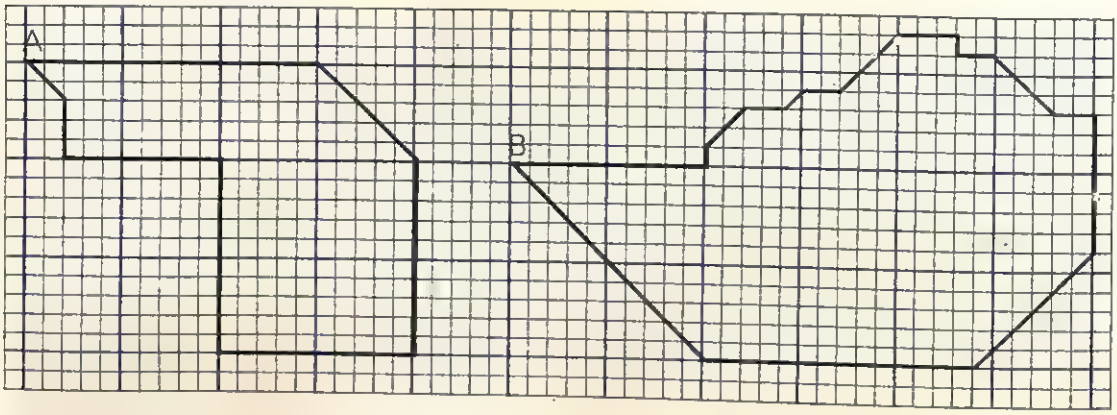
This is a drawing of a lawn with a flower bed in the middle.



Count the number of squares in the lawn area.

We can also use the squares on graph paper to measure areas.

Count the number of squares in each of these shapes.



Did you discover how many of the small squares were contained in each of the large squares on the graph paper?
Is it sufficient to tell someone that the area of a shape is a certain number of squares? We have already used squares of different sizes.

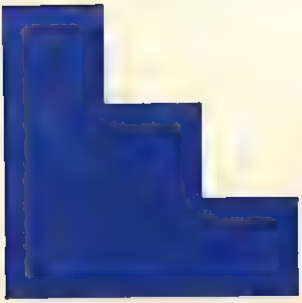
Small areas are usually measured in squares the sides of which are 1 inch long, like this one. We call this an inch square, and its area is 1 square inch.

How many of the large squares on the graph paper make an inch square?

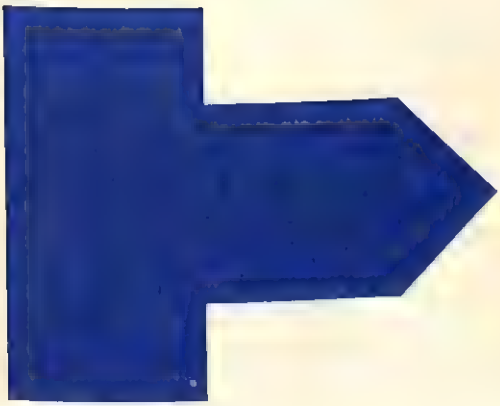
Find the number of square inches in each of the shapes on this page by drawing a network of inch squares on tracing paper and holding it over each shape in turn. You will have to estimate halves and quarters of an inch square.

A

B



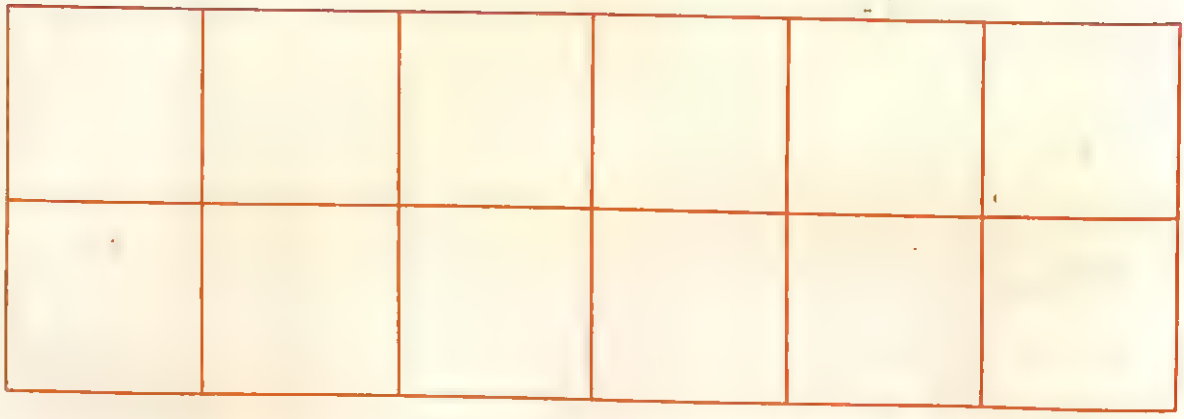
C



D

Areas of rectangles and squares

Count the number of inch squares in this rectangle.

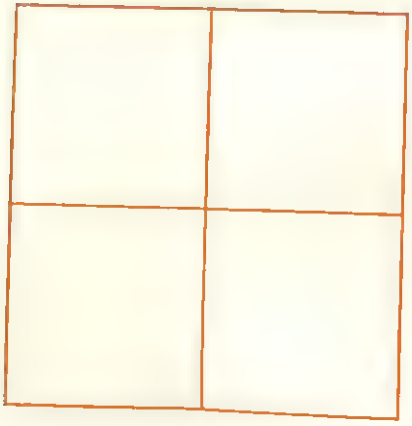


What is the quickest way of finding the total number of square inches?

Notice that there are 2 rows, with 6 squares in each row.

Let us call the number of squares in the length, l;
and the number of squares in the breadth, b;
and the area, a square inches.
Then $a = l \times b$.

Can we find the area of this square in the same way?



Check your answer by counting the number of inch squares.

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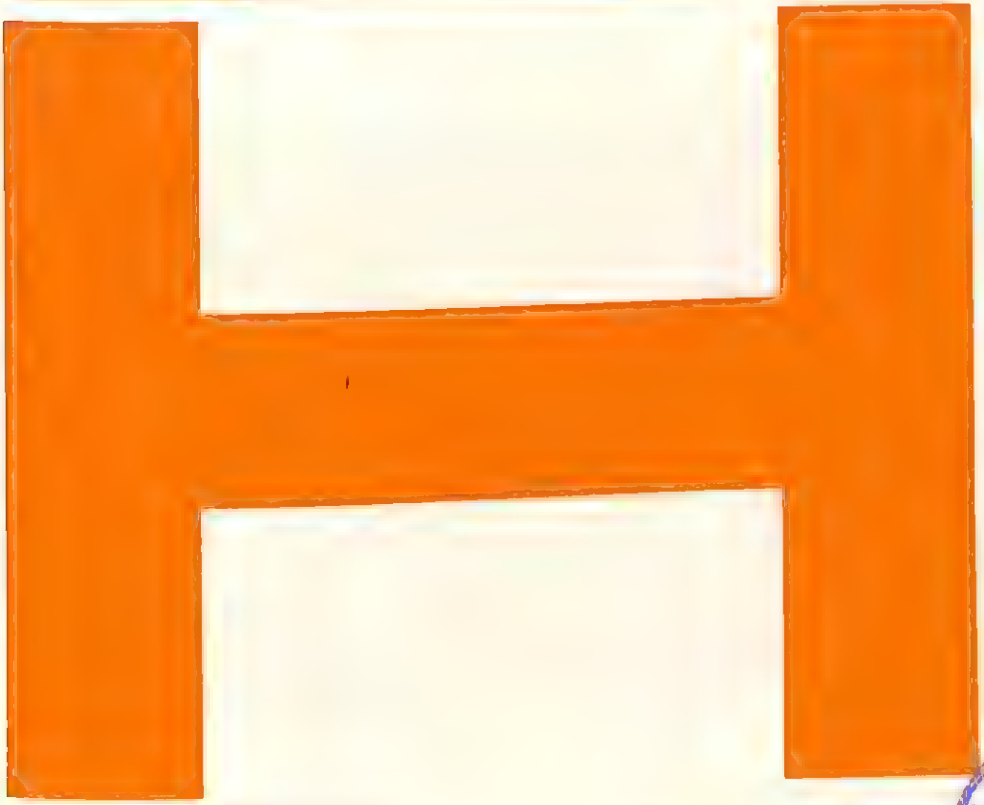
Find the area of these shapes in square inches.

A

B



Did you notice that shape B consists of three rectangles, and that two are alike?

We do not measure the area of a room in square inches.

Why not ? What size square do you think is suitable ?

Cut out a foot square. Use it to estimate the area of your desk, your teacher's desk, the blackboard, the door, a cupboard, or a window. Find out how many inch squares are contained in a foot square.

This is the plan of a classroom.

Find its area in square feet.

Find the area of your classroom. Ignore fractions of a foot when you measure the length and the breadth.

Perhaps your teacher will allow you to find the areas of other rooms and corridors.

When we measure the areas of fields, gardens, and building plots we usually use a bigger unit than the square foot.
What do you think it is ?

Draw a square yard in chalk on the floor or playground. Find out how many foot squares are contained in a yard square.

Large areas such as farms and parks are measured in acres. An acre is 4840 square yards, which is about the size of a football pitch.

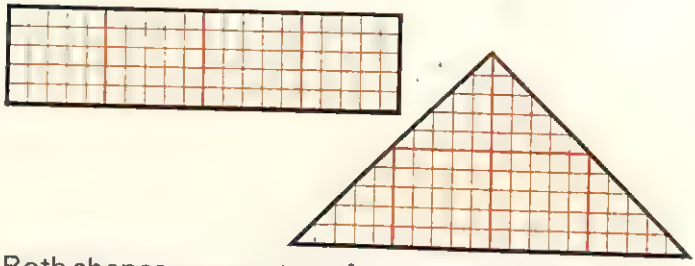
Ask your teacher or caretaker how many acres are occupied by your school buildings and playground. Look out for a notice like this :

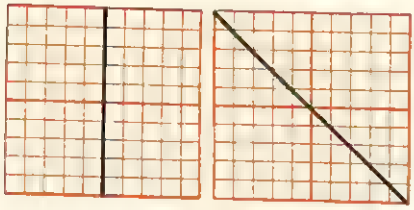
Even larger areas are measured in square miles. You will be able to find the area of your country in an encyclopaedia, or in the index of a large atlas.

Find the areas of the following countries and arrange them in order of size, the largest first :
France, Canada, Australia, Great Britain, Switzerland, Spain.

Equal areas in different shapes

Which of these shapes has the bigger area?
Think of a way to find this out.

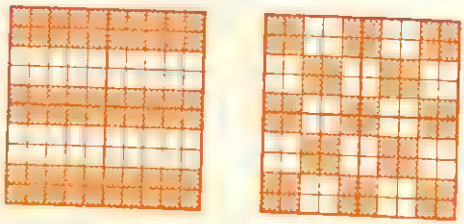




Both shapes were cut out from an inch square.
As we can see from the diagrams, the first square was divided into two rectangles and the second into two triangles.
The two rectangles have been joined together to make one large rectangle, and the two triangles have been joined into one large triangle.

Cut out an inch square and assemble it in the same way.

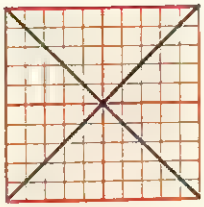
In these diagrams an inch square has been split up into different parts.



These parts can now be re-arranged like this.

All these shapes have the same area, 1 square inch.

Cut out an inch square from graph paper and join the opposite corners together like this.
Cut out the four triangles you have formed.
Re-arrange them to make different shapes.
Draw the shapes you make.
What is the area of each shape ?



Here are the shapes of two islands off the shores of Britain.
They are quite different in shape.

Find out from an encyclopaedia whether they are quite different in area.

Isle of Man

Isle of Anglesey

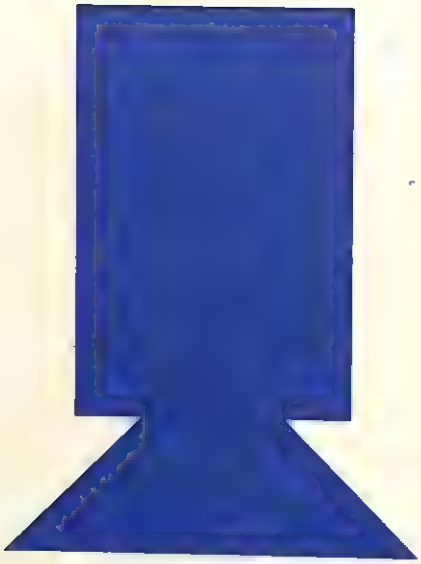
Cut out a square with sides 6 inches long from graph paper.
Divide it into seven parts like this :

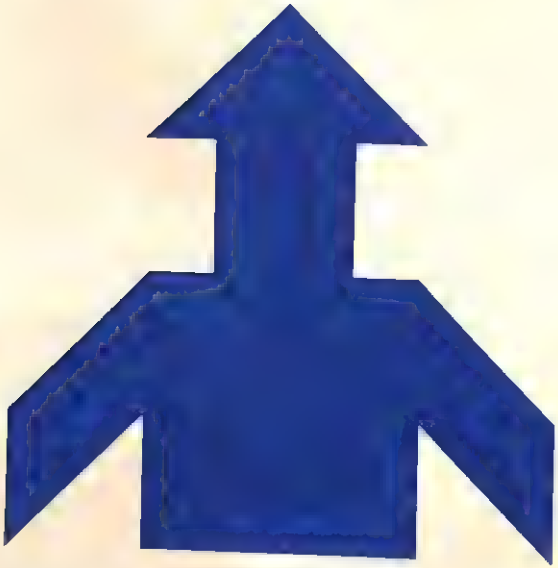
Cut out the seven parts.
Try to reassemble the square.
What is the area of the square ?

This is an old Chinese puzzle.

Can you make these shapes ?

What is the area of each shape ?





Try to make other interesting shapes by fitting the parts together.

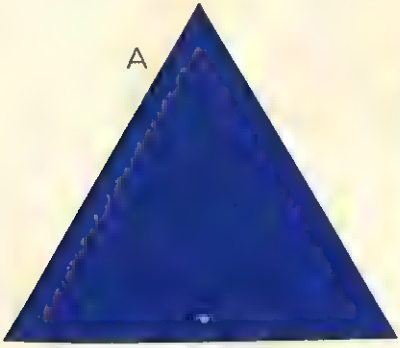
Perimeters

What is the distance all round this rectangle ?

We call this distance the PERIMETER.

Find the perimeter of each of these shapes.

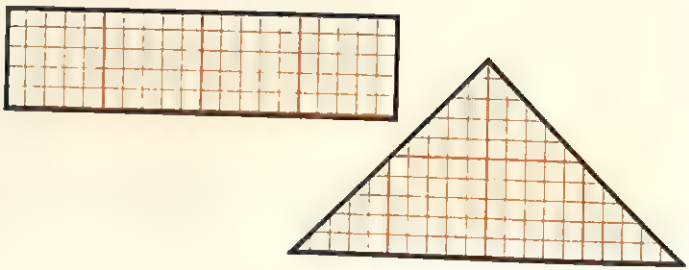
A



B

What is the perimeter of the inch square on page 25 ?
Find the perimeters of squares with sides 2", 3", and 4".
What do you notice about these perimeters ? How do they grow ?
How many times greater is the perimeter than the side of the square ?
What would be the perimeter of squares with sides 5", 6", 7", and 8" ?

Measure the perimeters of the rectangle and triangle opposite.
Is the perimeter for each the same as that of the inch square from which they were both made ?

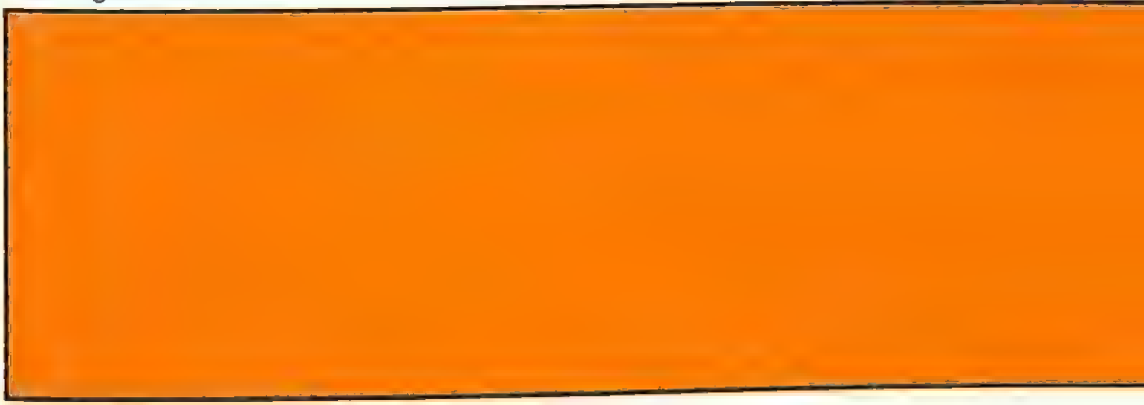


On page 25 you were asked to make shapes from the four triangles you cut out of an inch square.
Measure the perimeter of each of these shapes.
Are they the same ? Are they the same as the perimeter of the inch square ?

Have you discovered that two figures can have the same area but have different shapes and different perimeters ?

Now ask the question the other way round. Can different shapes have the same perimeter but different areas ?

Cut a piece of string or cotton exactly 16 inches long.
Arrange the string to make a rectangle like this.



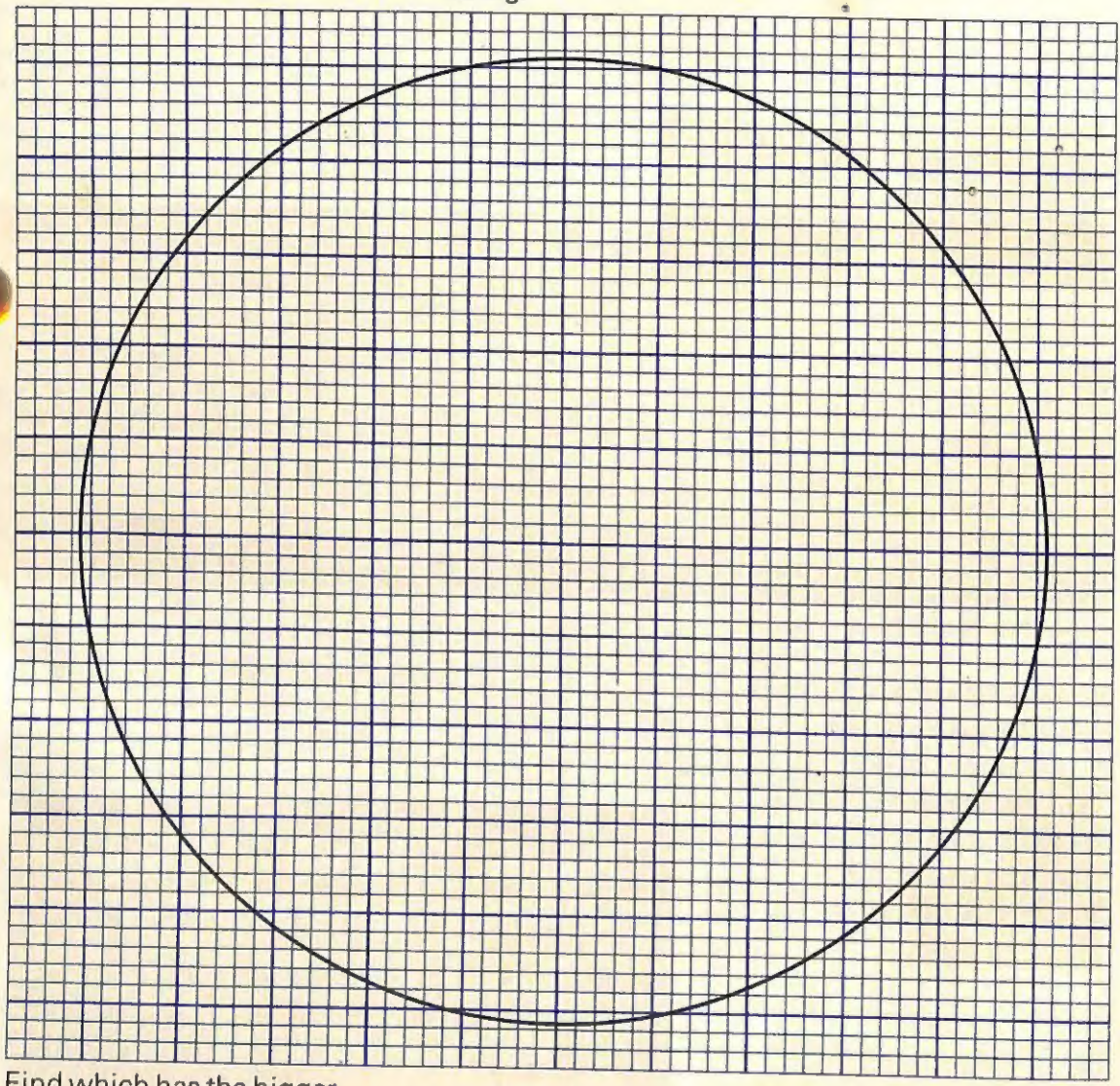
What is its area ?

Make rectangles of different shapes with the piece of string or cotton.
Find the length, width and area of each and write down what you find in a table like the one opposite.

LENGTH OF FIGURE	WIDTH OF FIGURE	AREA OF FIGURE

What is the greatest rectangular area you can make with the piece of string or cotton ?

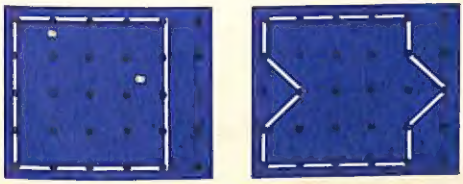
We have discovered that the same perimeter can enclose several areas. It can also be arranged to make several shapes.
Here is another shape enclosed by your piece of string.



Find which has the bigger area—the circle or the greatest rectangle you made.

Puzzle Pages

1 These two shapes have been made on the pin-board Which shape has the bigger area ?

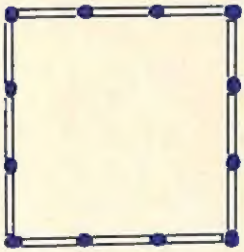


2 A man wanted to replace a broken tile in his kitchen. The tile was 1 foot square. Only 6" square tiles were left at the shop. The man bought two. Did he have enough ?

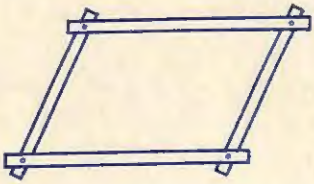
3 Show how to divide this letter L into four shapes making each the same size and shape.

4 Plastic material suitable for making a groundsheet for a scout tent costs 9d. per square foot. What would be the cost of making a groundsheet for the tent which is 12 feet long and 9 feet wide ?

5 Here are 12 matches arranged to make a square. The area of the square we will call 9 square 'matches'. Use the twelve matches to make the perimeter of a shape with an area of 5 square 'matches'.



6 Make a rectangle with four strips of card.
Is the area the same when you alter the shape to the one shown below ?
Is the perimeter the same ?
In which position will the strips enclose the greatest area ?



7 You have probably discovered that the greatest area which can be enclosed by a piece of string is a circle. If farmers had circular fields, would they need more or less fencing than for square fields?
Why is it that farmers do not have circular fields?

8 Suppose that you had two pieces of string of the same length. With the one you made a square and with the other a hexagon. Which shape would have the bigger area?

9 Find the area of this shape in: (a) square inches; (b) square feet.

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The author has prepared these books with the help and advice of some of the members of the School Mathematics Project who have been introducing new ideas and new spirit into mathematics in the early years of the secondary school course. They feel that some of these ideas and approaches are relevant to children in the primary schools; such children would be thereby more readily equipped for the pattern of the secondary school course which is now emerging.
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